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MHF4U Final Exam Review

Short Answer

1. Under what condition does a rational function have an oblique asymptote?

2. What are the zeros for the function $f(x) = \dfrac{(x-9)(x+2)}{x(x-1)}$?

3. What are the vertical asymptotes of the function $f(x) = \dfrac{(x-2)(x+5)}{x(x-5)}$?

4. What is the domain of the function $f(x) = \dfrac{x(x+1)}{(x-1)(x^2+4)}$?

5. What are the x-intercepts of the function $f(x) = \dfrac{x(x-3)(x+5)}{x-4}$?

6. What is the y-intercept of the function $f(x) = \dfrac{x^2-2x+1}{x+3}$?

7. Draw the graph of a rational function that has a vertical asymptote at $x = 3$ and a horizontal asymptote at $y = 2$.

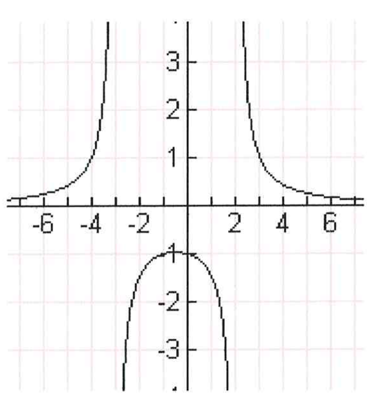
8. What are all the asymptotes of the function $f(x) = \dfrac{x+2}{x-3}$?

9. What is the domain of the function $f(x) = \dfrac{x^2-1}{(x-1)(x+2)}$? How many vertical asymptotes does this function have?

10. Create a rational function with vertical asymptotes at $x = 5$ and $x = -3$, a horizontal asymptote at $y = 0$, and an x-intercept of 2. Describe your process in creating the function.

11. Find the equations of all asymptotes for the rational function $f(x) = \dfrac{x^2+3x-2}{x+2}$.

12. Find the equation for the following graph in the form $f(x) = \dfrac{k}{x^2+bx+c}$.



13. For the rational function $f(x) = \dfrac{x^2-4}{x^2-3x+2}$, determine the equations of all asymptotes.

14. Create a rational function that has a domain of $\{x \mid x \neq -2, 7, x \in R\}$ and a vertical asymptote at $x = 7$. Describe the graph of this function.

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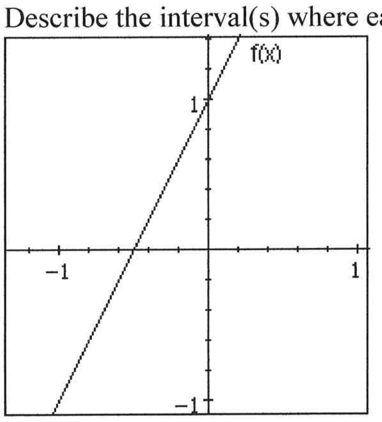
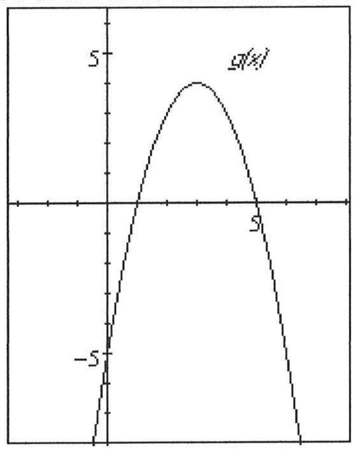
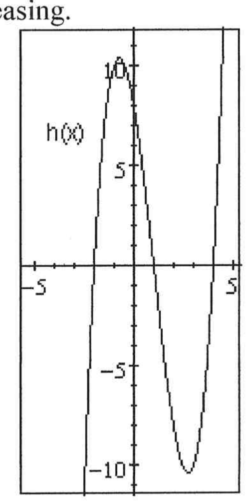
15.

Using domain, intercepts, and asymptotes, draw the graph of $f(x) = \dfrac{x-5}{x^2+2x-15}$.

16. Describe the interval(s) where each of the three functions is increasing.

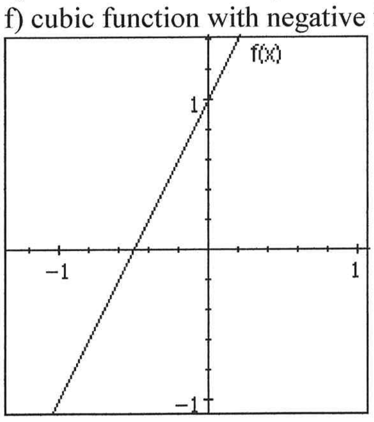


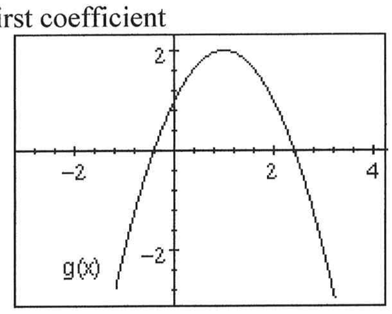


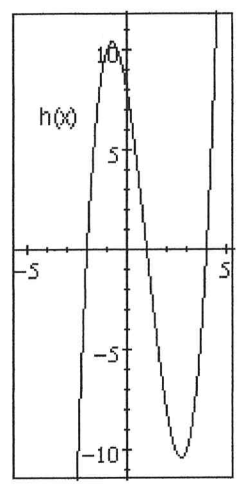


17.

Match each graph with a description.
a) increasing linear function
b) constant linear function
c) quadratic function with a local minimum
d) quadratic function with a maximum
e) cubic function with positive first coefficient
f) cubic function with negative first coefficient







18.

Describe the difference between a local maximum and a local maximum value.

19. Describe the behaviour of a function around a local minimum point.

20.

Write $f(x) = -2(x-1)^2(x+3)$ in expanded form.

21.

Sketch the graph of $f(x) = (x-3)(x+2)(x+5)$ using the zeros and end behaviours.

22.

Determine the remainder r for $(x+1)(x-2) + r = x^2 - x + 7$.

23.

a) Divide $x^4 + x^3 + 2x^2 + 1$ by $x + 1$ using synthetic division.
b) Write the division statement.

24.

Is $x - 1$ a factor of $x^3 + 2x^2 - 6x + 3$?

25. Determine the remainder when $x^3 + 2x^2 - 6x + 1$ is divided by $x + 2$.

26.

Use long division to divide $2x^3 + 5x^2 - 4x - 5$ by $2x + 1$.

27. Use synthetic division to divide $2x^3 + 5x^2 - 4x - 5$ by $2x + 1$

28. Factor $x^3 - 5x^2 - x + 5$.

29. Factor $4x^3 + 4x^2 - x - 1$.

30. Factor $x^3 - 1000$.

31. Factor $x^4 - 7x^2 - 6x$ fully.

32. Solve $x^3 - 3x^2 - x + 3 = 0$.

33. Algebraically solve $x(2x + 1)(x - 4) > 0$.

34. Determine the equation of the function with zeros at ± 1, -2, and y-intercept of -6. Sketch the function using this information.

35. a) Without dividing, calculate the remainder when $4x^4 - x^3 + 2x^2 - 1$ is divided by $2x - 1$.
b) Is $2x - 1$ a factor of $4x^4 - x^3 + 2x^2 - 1$?

36. Solve $x^3 - 3x^2 + 5x - 3 = 0$.

37. The mass of a radioactive substance decreases to half its value every 125 days. What would be the mass of a 500 g sample after 2 years?

38. What is the equation of the asymptote to the curve $y = 3(2)^x - 2$?

39. What is the y-intercept of the function $y = 3(2)^x - 4$?

40. What transformation changes the graph of $y = 3(2)^x$ into $y = 3(2)^x - 4$?

41. What transformation changes the graph of $y = (2)^x$ into $y = 3(2)^x$?

42. State the intercepts of the logarithmic function $y = \log_a x$.

43. What single transformation of the graph of $y = a^x$ would produce the graph of $y = \log_a x$?

44. How are the exponential and logarithmic functions related?

45. Evaluate: $\log_2 32$

46. Evaluate: $\log_4 \frac{1}{64}$

47. Evaluate: $\log_9 9$

48. Evaluate: $\log_3 108 + \log_3\left(\frac{3}{4}\right)$

49. Write $3\log_b 2 - 2\log_b 3 + 4\log_b 2$ as a single logarithm in exact form.

50. Solve for x: $3\log_4 x = \log_4 125$

51. Solve for x to two decimal places: $2^x = 29$

52. Solve for x to two decimal places: $-8 \times 2^x = -79$

53. Solve for x to two decimal places: $125 = 15\left(2^{\frac{5}{x}}\right)$

54. Graph the function $y = 3(2)^x - 4$ and state the domain, range, equation of the asymptote, and the y-intercept.

55. Graph the function $y = (\frac{1}{2})^x + 3$ and state the domain, range, equation of the asymptote, and the y-intercept.

56. Without graphing, determine the domain, range, y-intercept, and equation of the asymptote of the function $f(x) = 2(3)^x + 7$.

57. Sketch the graph of $y = 2^x$ and show how to use it to sketch the graph of $y = \log_2 x$.

58. Evaluate: $\log_8 6 - \log_8 3 + \log_8 4$

59. Solve for x: $\log_4(x-1) - \log_4(x+2) = 1$

60. Solve for x: $\log_3 x + \log_3 3 = \log_3 1 + \log_3 4$

61. The pH scale is a logarithmic scale for measuring the acidity of a chemical solution. It is calculated using $\text{pH} = -\log[H^+]$, where $[H^+]$ is the molar concentration of hydrogen ions in the solution. If the pH of a solution changes from 2 to 8, by what factor does the hydrogen ion concentration change?

62. A bacteria colony grows exponentially from 134 cells to 1241 in 24 hours. What is the doubling period in hours?

63. The table below is used to find the instantaneous rate of change of $f(x) = 36x^2 - 24x + 4$ at the point (1, 16). Use the table to find the best estimate for this value.

Interval	$\Delta f(x)$	Δx	**Average Rate of Change,** $\frac{\Delta f(x)}{\Delta x}$
$1 \le x \le 1.5$			
$1 \le x \le 1.1$			
$1 \le x \le 1.01$			
$1 \le x \le 1.001$			

64. For $f(x) = ax^2 + bx + c$, determine the average rate of change of $f(x)$ with respect to x over the interval $0 \le x \le 3$.

65. The position of a particle is given by the function $s(t) = -15 - 9.8t$. Find the average rate of change in distance with respect to time over the first 8 seconds.

66. The position of a particle is given by the function $s(t) = 100 - 15t - 4.9t^2$. Estimate the particle's instantaneous rate of change of distance at 1 s.

67. The position of a particle is given by $s(t) = t^2 - 4t + 4$, where s is measured in metres and t in seconds. Find the instantaneous rate of change of distance at 2 s.

68. An approximate model for the distance s metres traveled in t seconds by a skateboard going down a hill is $s = kt^2$, where k is a constant that depends on the slope of the hill. If, for one particular hill, the value of k is 0.1, estimate the instantaneous rate of change of distance with respect to time at 10 seconds.

69.

A particle moves along a straight path and its position s (in cm) at t seconds is given by $s(t) = \dfrac{t}{2t^2 + 1}$.

Estimate the instantaneous rate of change of the particle at 3 seconds?

70.

A flying squirrel took off from a branch and landed on the branch of another tree 6 metres lower. Hint: The function $d(t) = 4.9t^2$ models the distance d in metres the squirrel has travelled after exactly t seconds pass.

Interval	Δd	Δt	**Average Rate of Change,** $\dfrac{\Delta d}{\Delta t}$
$5 \le t \le 6$			
$5.5 \le t \le 6$			
$5.9 \le t \le 6$			
$5.99 \le t \le 6$			

a) Copy and complete the table.
b) Estimate how fast the squirrel was falling when it reached the second branch.

71.

Let $f = \{(-2, 2), (-1, 1), (0, 2), (1, 3), (2, 4)\}$ and $g(x) = \sqrt{x-1}$. Determine the domain of $f \circ g$.

72.

Let $f = \{(-2, 5), (0, 7), (2, 9)\}$ and $g(x) = \sqrt{x+2}$. Determine the domain of $(f \circ g)(x)$.

73. Let $f(x) = 4x + 1$ and $g(x) = (3 - 2x)^2$. If $h(x) = (g \circ f)(x)$, evaluate $h(2)$.

74.

Let $f(x) = 3x^2 + 2$ and $g(x) = 5 - 3x$. Determine $(g \circ f)(2)$.

75.

Let $f = \{(2, 3), (4, 5), (6, 7)\}$ and $g = \{(1, 2), (2, 4), (3, 8)\}$. Determine $g \circ g^{-1}$ and $f \circ g$.

76. Let $f(x) = 2x + 3$. Determine $\left(f \circ f^{-1}\right)(x)$.

77. Let $f(x) = x^2 + 3x$ and $f(g(x)) = x^2 + 7x + 10$. Determine $g(x)$.

78.

Bill earns \$30/h as a plumber plus \$23/d for travelling expenses. He pays 1.5% of his daily earnings for union dues.
a) Write functions that model his daily earnings and his daily union dues.
b) Draw an input-output diagram to illustrate how the union dues are related to earnings for an 8 h day.
c) How much are the union dues for an 8 h day?

79.

a) Let $f(x) = 3x - 2$ and $g(x) = x^2$. Determine $(f \circ g)(x)$.

b) Graph f, g, and $(f \circ g)(x)$ on separate axes and again on the same axes.

c) Describe the graph of $(f \circ g)(x)$ as a transformation of the graph of $g(x)$.

80. If sin X $= -\dfrac{1}{2}$, $0 \le \theta \le 2\pi$, find the possible measures of $\angle$X.

81. Change each degree measure to radian measure (or radian measure to degree measure). Round to the nearest hundredth of a radian, if necessary.

a) 210° **b)** 135° **c)** 32.4° **d)** $\dfrac{\pi}{5}$ **e)** $\dfrac{11\pi}{6}$ **f)** 0.8

82. Find the exact value of each trigonometric ratio.

a) $\tan \dfrac{2\pi}{3}$ **b)** $\cos \dfrac{7\pi}{6}$ **c)** $\sin \dfrac{5\pi}{6}$ **d)** $\cos \dfrac{2\pi}{3}$

83. Sketch one cycle of the graph of each of the following. State the domain, range, amplitude, period, and phase shift of the cycle.

a) $y = \sin \frac{1}{2}(x-\pi) - 3$ **b)** $y = 3\sin\left(\frac{1}{3}x - \frac{\pi}{9}\right) + 2$ **c)** $y = 2\sin x + 1$ **d)** $y = \cos \frac{1}{2}(x-\pi) - 3$ **e)** $y = -4\cos 3\left(x - \frac{\pi}{4}\right)$ **f)** $y = 3\sin x + 1$ **g)** $y = 2\cos\left(x - \frac{\pi}{4}\right)$

h) $y = 0.5\sin 3\left(x + \frac{\pi}{2}\right)$ **i)** $y = 5\cos\left(\frac{1}{3}x - \frac{\pi}{6}\right) - 2$

j) $y = -2\sin(x-\pi)$

84. a) Sketch the graph of y = sin x and its reciprocal function.
b) State the key features of each graph, including domain, range, period, key points and asymptotes.

85. Solve each equation for $0 \le x \le 2\pi$. Express your answer to 2 decimal places if necessary.

a) $2\sin x + \sin x \cos x = 0$
b) $2\tan^2 x + 3\tan x + 1 = 0$
c) $4\sin x \cos x - 2\sqrt{3}\sin x + 2\cos x - \sqrt{3} = 0$
d) $2 + \cos x = 2\cos x + 3$
e) $2\cos x + \sin x \cos x = 0$
f) $3\sin^2 x + 7\sin x + 2 = 0$
g) $2\cos\theta - \sqrt{3} = 0$
h) $7\sin\theta = 5\sin\theta - 1$
i) $4\sin^2 x + 2\sin x - 2 = 0$
j) $5\sin^2 x - 18\sin x - 8 = 0$
k) $4 - 4\cos x = 4\sin^2 x - 1$

86. Prove each identity.

a) $\frac{1}{\cos x} + \tan x = \frac{\cos x}{1-\sin x}$

b) $1 - 2\tan x + \tan^2 x = \frac{1 - 2\sin x\cos x}{\cos^2 x}$

c) $\frac{\cos^2 x}{1 + 3\sin x - 4\sin^2 x} = \frac{1+\sin x}{1+4\sin x}$

d) $\frac{\cos x - \sin x}{\cos^2 x} = \frac{1-\tan x}{\cos x}$

e) $\frac{\sin x}{\cos^2 x\tan x} = \frac{1}{\cos x}$

f) $\sin x = \frac{1}{\sin x} - \frac{\cos x}{\tan x}$

g) $\frac{\sin^2 x + \cos^2 x}{1+\cos x} + \frac{1}{1-\cos x} = \frac{2}{\sin^2 x}$

h) $\frac{\cos A - \sin 2A}{\cos 2A + \sin A - 1} = \cot A$

87. A small windmill has its centre 7 m above the ground and blades 2m in length. In a steady wind, point P at the tip of one blade makes a complete rotation in 16 seconds. The height above the ground, h(t), of point P, at the time t can be modeled by a cosine function.

a) If the rotation begins at the highest possible point, graph two cycles of the path traced by point P.

b) Determine the equation of the cosine function.

c) Use the equation to find the height of point P at 10 seconds.